

The Sixth Conference on Information Theory and Complex Systems
TINKOS 2018

BOOK OF ABSTRACTS

Editors: Velimir Ilić and Miomir Stanković



Belgrade, Serbia, June 18-19, 2018
Mathematical Institute of the Serbian Academy of Sciences and Arts

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МИНИСТАРСТВО ПРОСВЕТЕ
НАУКЕ И ТЕХНОЛОШКОГ
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THEMATIC FIELDS

Information theory
Information transmission
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Intelligent systems
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Mathematical physics

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The Traditional History in Terms of Complex Systems

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Keywords

time operator; multiresolution; p-adic numbers; philosophy of history; iconography

Summary

I had a feeling once about Mathematics, that I saw it all — Depth beyond depth was revealed to me — the Byss and the Abyss. I saw, as one might see the transit of Venus — or even the Lord Mayor's Show, a quantity passing through infinity and changing its sign from plus to minus. I saw exactly how it happened and why the tergiversation was inevitable: and how the one step involved all the others. It was like politics. But it was after dinner and I let it go! The practical point is that if this aged, weary-souled Civil Service Commissioner had not asked this particular question about these Cosines or Tangents in their squared or even cubed condition, which I happened to have learned scarcely a week before, not one of the subsequent chapters of this book would ever have been written. I might have gone into the Church and preached orthodox sermons in a spirit of audacious contradiction to the age. I might have gone into the City and made a fortune.

– Winston Churchill [1]

The concept of a negative number emerges in European mathematics from the XV century onwards within the context of algebraic calculations. However, the order relation between positive and negative integers remained unclear for a long time, giving rise to a discussion in which some of the great mathematician considered the negatives to be greater than infinity. The statement occurred in *Arithmetica infinitorum* of John Wallis, being also supported by Leonard Euler and Blaise Pascal [2]. At the very best, it is about an alternative order relation and at the worst, about an alternative definition of the number.

The author elaborates the issue in terms of the time operator formalism developed by Ilya Prigogine concerning the complex systems physics [3]. Supposing that the system evolves by the group action $U^\tau f_t = f_{t+\tau}$, the time operator $Tf_t = tf_t$ satisfies the uncertainty relation $[T, U] = U$. It is considered to be a straightforward generalization of multiresolution which is a core of the number system [4].

A p -based number system is represented by the module $\mathbb{Z}(p)$ consisting of formal series $\sum_n a_n p^{-n}$, whereat each subspace \mathcal{D}_n – whose elements are $a_n p^{-n}$ – corresponds to details at a resolution scale n . The detail subspaces are interrelated by a shift property $\mathcal{D}_{n+1} = U(\mathcal{D}_n)$, which is done by the operator $Uf = f/p$ generating an evolutionary group. The time

operator is given by $Tf = -p\partial_p f$, whereat ∂_p signifies formal differentiation of a power series. The uncertainty relation is satisfied since

$$[T, U] = -p\partial_p \frac{\blacksquare}{p} - \frac{-p\partial_p \blacksquare}{p} = \frac{\blacksquare}{p} = U.$$

For each $f \in \mathcal{D}_n$, it holds $Tf = nf$ meaning that \mathcal{D}_n s are eigenspaces of the operator. The time succession relates innovations in the system concerning the resolution increase. Thereby $\mathcal{A}_m = \sum_{n \leq m} \mathcal{D}_n$ are approximative subspaces of the multiresolution [5], consisted of the series $\sum_{n \leq m} a_n p^{-n}$. Restricting to the basic case $p = 2$, one gets the homomorphism $h: \sum_{n \leq m} a_n p^{-n} \mapsto \sum_{n \leq m} a_n 2^{-n}$ of the subspaces \mathcal{A}_m into $\mathcal{A}_m^{(2)}$ that are the approximative subspaces concerning multiresolution of the dyadic numbers $\mathcal{A}^{(2)} = \bigcup_m \mathcal{A}_m^{(2)}$. The detail subspaces of the multiresolution are given by $\mathcal{D}_m^{(2)} = \mathcal{A}_m^{(2)} \setminus \mathcal{A}_{m-1}^{(2)}$, defining the dyadic log-norm $\|\blacksquare\|^{(2)}$ on $\mathcal{A}^{(2)} \setminus \{0\}$ through $\|x\|^{(2)} = m \Leftrightarrow x \in \mathcal{D}_m^{(2)}$.

The evolutionary operator $U^{(2)}x = x/2$ is a projection of U through the homomorphism h meaning that $U^{(2)} \circ h = h \circ U$. The intrinsic time of $\mathcal{A}^{(2)}$, satisfying the uncertainty relation, is the operator $T^{(2)}x = \|x\|^{(2)} \cdot x$. However, it is not a linear operator since the dyadic numbers do not have any module structure.

$\mathcal{A}^{(2)}$ is the field [6], respecting the operations $+$ and \cdot defined at the series. An element $\sum_{n \leq 0} a_n 2^{-n}$ of $\mathcal{A}_0^{(2)}$ is representable in the form $\cdots a_{-2}a_{-1}a_0$ whereby addition and multiplication are defined from right leftwards. The same holds for elements of $\mathcal{A}_m^{(2)}$, $m > 0$, that are presented by m binary digits more $.a_1a_2 \cdots a_m$ continuing the string rightwards from the point on.

Subtraction and division are defined to be the inverse operations of addition and multiplication, respectively.

The primary significance of dyadic system is a capability to represent the negative numbers. A positive integer (including zero) is represented in the form $+x = \dots 000 \dots a_{-2}a_{-1}a_0$ consisting of digits that are all zero from a position leftwards. Its additive inverse corresponds to $-x = \dots 111 \dots \widetilde{a}_{-2}\widetilde{a}_{-1}\widetilde{a}_0 + 1$, whereby $\widetilde{a} = \begin{cases} 1, & a = 0 \\ 0, & a = 1 \end{cases}$ is the complement of a binary digit. One

proves $-x + x = 0$ following definition of the addition operation. According to that, the representation of negative integers consists of binary digits which are units from a position leftwards. Lexicographic order \preceq of the dyadic numbers is a transfer of the common order relation on both positive and negative integers apart. However, it is a partial order on $\mathcal{A}^{(2)}$, implying that all positive integers are less $<$ than the negative ones.

The multiplicative inverse of a nonzero element is also unique in the field. For example $3 = \dots 00011$ inverts to $1/3 = \dots 101011$, which is confirmed by multiplication $3 \cdot 1/3 = 1$. Similarly, each rational number is represented in $\mathcal{A}^{(2)}$ by a string that is periodic from a position leftwards. In that respect, \mathbb{Q} is a subfield of $\mathcal{A}^{(2)}$.

Non-periodic strings correspond to irrational dyadic numbers. However, they are not generally interpretable in \mathbb{R} and therefore best considered to be some infinities since representing the sums of divergent series. Respecting lexicographic order, they are greater than all strings started by the period 0 and less than all started by the period 1. According to that, the negative integers are greater not only than the positive ones, but also than the irrational infinities. On the other hand, the positives are less than both of them. The same holds for p -adic numbers $\mathcal{A}^{(p)}$ wherein p is a prime [6].

A significance of the p -adic numbers in interpretable in terms of historical conceptions. The modern concept refers to the real numbers \mathbb{R} that is a complete, linearly ordered field [7]. The time corresponds to a succession of non-lasting elements, each of them regarded to be a date. In that respect, the linear order \leq represents a temporal relation establishing the real line whereat the negative elements precedes the positive ones. Therefore, time dates from the central position signified by zero in both positive and negative directions. The linear time of real numbers is therefore undirected, meaning that it dates both directions equally starting from the center. Just because of that, the number is required to include a \pm sign. Before the XVIII century, there is no such a historical conception [8], but it coincides to an emergence of the mathematics considering negative numbers to be less than zero. An alternative view, that concerns p -adic numbers, corresponds to the traditional history.

The traditional concept implies an intrinsic time of the history that relates its evolution. It corresponds to the p -

adic log-norm, making $\mathcal{A}^{(p)}$ also a complete field in regard to that. Although the relation \preceq is a partial and not a linear order, it considers the negative numbers to be greater than the positive ones. The order is thereby an invariant of the evolution that does not concern the translation over the dates, but a dilatation of the structure. The dates are starting from zero and expanding in a single one direction, given by the evolutionary dilatation. The traditional history is therefore directed, due to an inherent expansion of $\mathcal{A}^{(p)}$. The dates start with a creation of the world, having ended with its completion. The concept of liturgical time, that is considered not to be a linear order, but the continual presence of both past and future, is aligned to such a philosophy of history. It is reflected by the Byzantine style iconography that is an original expression of the liturgical tradition [9].

The liturgical time outlines dynamical spatiality, that is regarded to be a defining feature of the icon [10]. Such a geometry has been considered from the viewpoint of the complex systems physics and fractal design [11]. However, p -adic numbers have never been used in that regard which is a significant potential for its elucidation.

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Paradigm of Complexity as a Framework for Philosophical Synthesis of Knowledge Management

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Keywords

Complexity; Knowledge; People management practice; Dialogue; Organic system

Summary

There is an opinion that two forms of complexity exist. One of them is “restricted” and interested essentially in complex dynamical systems and it constructed its own field within science. The other one is “generalized” complexity that involves all fields and, moreover, relates to our knowledge as human beings [1].

Today, there is a need for creating strategies in the people management practices, which should be based on complexity postulates that refer to collective learning and management of knowledge [2]. Those practices are rooted in disciplines such as behavioral sciences, human resource management, business psychology etc. Furthermore, there is a need for multidisciplinary approach because of the evidence that some processes underlie the collective human behaviour which could be studied in terms of complex networks theory [3].

Some of the theorists come to the idea of need for contextualization, and some of those concepts are recognized in the field of organizational studies [4]. Some of the scholars claim that organizations should be regarded rather as open, dynamic, organic systems than connected components with mechanical usage rules [5]. So, there is a voice from within a management practice that we should be moving from a mechanical to biological approach and that this should be done by discovering a new philosophical synthesis [6].

One may think that complexity is already recognized as a framework for such a synthesis in different fields of human creativity, from art towards science [7] and management.

There is a notion that the knowledge creation should be conceptualized as a dialectical

process, in which various contradictions are synthesized through dynamic interactions among individuals, the organization, and the environment [8].

So it looks like that the theoretical physicist David Bohm did something ingenious when he developed a team dialogue method where the team opens up to the flow of the “higher intelligence” [9]. This is also known in philosophical concepts claiming that a whole is always more than a sum of its simple components.

It is supposedly so because these „components“, the individuals establish mutual dynamic relationships and these relationships generate new quality that pervades the whole. But sometimes, due to the constraints, restrictions and servitudes in organization, a “whole “ makes the components and the individuals lose their qualities or they inhibit them [10]. Hence, this question arises: “How to release the human potential within an organization at all levels?” This should be achieved throughout the purposeful, conscious communication [11]. There are some research studies showing that organizations where the quality of interpersonal relationships has been better nourished – generate bigger profit.[12] Thus, what is intangible by its essence is manifested in the material world.

It is clear now that the “state of mind” has its influence of our techno – economic development and it is also manifested in the biosphere [13]. Complexity paradigm, thus, has the role to introduce a different approach not only in human resource management, economics and social dynamics, but also in ecology, evolutive biology, anthropology and cosmology [14].

New philosophical synthesis should connect “restricted” and “generalized” complexity in order to reform all of the ways of knowing and thinking [15].

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Juncture between Complex Theory and History & Philosophy of Science

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Keywords

Complexity; History and philosophy of science; Ptolemaic system; Scientific revolution; Complex light

Summary

Complexity research shows certain trends, that include: interdisciplinarity, unexpected fields and humanities [1]. At current stage of the theory of complex systems certain historically and philosophically recognizable situations and philosophic-scientifically re-examination which refers to the history of science, appear. As a presentation of that, we introduce two examples: the first concerns the interpretation of a well known situation from the history of science in the context of determining the origin of complexity, and the second reflects the current state in research of light and photonics.

Beside high dimensionality, network interactions, and nonlinearity which are commonly thought to be origins of complexity, in his search for additional roots for complexity Schuster points out more reasons for which a problem may appear complex: the lack of knowledge to comprehend the problem, deficiency of methods to interpret it adequately, and embedding of a simple system in a complex environment [2].

A situation from the history of astronomy that can be seen as the lack of knowledge which is the origin of complexity, is development of a system of the world [2]. The Pythagorean system of concentric harmonic spheres (6 BCE) didn't fit the astronomical data [2]. Contrary to that, Ptolemaic system (2 AD) represents a mathematical description of astronomical data from many centuries. The system was complex with a tendency to increase the number, inclination and function of constructs such as epicycle, deferent, equant and eccentric in order to keep celestial bodies moving on perfect cycles with uniform angular velocity [2]. Such a way of describing motions in astronomy can be compared to giving explanations by using complex systems [2]. Copernicus's heliocentric system (Mikolaj Kopernik, 1473-1543), Kepler's laws of planetary motions (Johannes Kepler, 1571-1630) and Newton's laws of mechanics (Sir Isaac Newton, 1643-1727) simplified the system of the world

explanation and made a revolution in science [2]. The alternative idea and the discovery of universal laws led to a simple description of the planetary motions [2].

Our second example is taken from the domain of electromagnetic waves, whose discovery Popper (1902-1994) considers a revolution that is greater than Copernican [3]. It concerns the current state of the research of light and photonics. Since the discovery of electromagnetic waves and Maxwell's equations (James Clerk Maxwell, 1831-1879) which describe the classical electromagnetic field, the applied knowledge about electromagnetic waves has made a great impact on everyday life [4]. New ground-breaking advances and new stage in technological development are expected now from the highly active interdisciplinary field of complex light [4]. Complex light includes fundamental points some of which are beams with a structured wavefront, classical and quantum aspects of the spin and orbital angular momentum of light and novel propagation dynamics. Laguerre-Gauss, Hermite-Gauss, Bessel, Mathieu, Airy, helico-conical beams; imaging with structured light; quantum information processing and imaging with complex light; entanglement and hyper-entanglement with spatial modes are some of the included topics, too. In modern photonics paradigm shifts in magnetism at optical frequencies, backward waves, "engineering" of space and light have taken place [5]. New paradigm is an information-driven imaging [6]. In research of light and photonics complexity has become a method for solving puzzles but also for breakthroughs. According to modern optics, light can be a more complex phenomenon then it was previously considered [4].

Philosophical analysis at first appears with the question: Are the considered cases complex or complicated?

In claims that a new paradigm [5] or fundamental breakthrough [4] is emerging, we face ourselves with philosophy of science. In Kuhn's theory (Thomas Kuhn, 1922-1996) the position in which light research and photonics are, could be a puzzle solving within normal science, or a crisis which takes place in the

expectation of scientific revolution [7]. For that reason factors which may lead to a new revolution, such as a different approach, are being investigated [8].

The emerging science of complexity “looks set to trigger the next great wave” [9] from which history and philosophy of science are inseparable.

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Centrosome Frequency Affects Energy of Pairs of Oscillating Chromosomes

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Keywords

Mitotic spindle; Complex system; Centrosome frequency; Mechanical energy; Mechanical model

Summary

Centrosome is a complex structure that functions as a microtubule organizing center during mitosis. Its oscillatory movements during mitosis affect the polarity of the mitotic spindle [1]. Centrosome origin and its position during cell division process affects the stability of the mitotic spindle and contributes to asymmetric cell division in some cells [2]. The oscillating frequencies of centrosomes could change during some physiological processes and during mitosis [3].

The aim of this work was to study how different centrosome excitation frequency affects the energy of pairs of homologue chromosomes in the system of mitotic spindle during metaphase. The analyses were done through mechanical oscillatory model of mitotic spindle [4].

The oscillatory behavior of this model is based on dynamics of coupled systems [5], mitotic spindle is presented as a system of coupled oscillators [4]. Each element in the model has its mechanical counterpart. Centrosomes are presented as mass particles that represent two rheonomic centers of oscillations. [4].

Assumptions of the model: rheonomic centers of oscillations-centrosomes oscillate with single frequency, system of mitotic spindle is linearized, angle of mitotic spindle was taken as $\pi/2$, and chromosomes with heavier masses are positioned in the central part of metaphase plate. Numerical analyses for kinetic, potential and total mechanical energy of the first pair of mouse chromosome were done for different values of centrosome oscillatory frequencies. Frequency of centrosome oscillations was calculated according to the data for angular frequency. Data for numerical analysis (chromosomal mass, rigidity of eukaryote metaphase chromosomes, rigidity for microtubules at 37° C, centrosome mass, amplitude oscillations and circular frequency) were taken from the literature [4].

Our results show that centrosome frequency change can change energy of the same homologue pair of chromosome when it remains in the same position in mitotic spindle indicating that centrosome frequency change can change energy code of the chromosome pair. Changing the frequency of centrosome oscillations induces the phase shift in kinetic and potential energy curves of the same oscillating homologue chromosome pair. Besides, kinetic energy of the same chromosome pair shows amplitude change with centrosome frequency change.

This could be of importance for process of cell differentiation.

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Generalized function of fractional order dissipation and independent fractional type modes of a class of discrete system oscillations

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Keywords

Generalized function of fractional order dissipation of energy, standard light fractional order mechanical element, standard light fractional order electrical element, eigen main independent fractional order mode, eigen main nonlinear fractional order mode, eigen main fractional order mode energy dissipation, fractional order discrete system, theorem of mechanical energy change, qualitative and mathematical analogies, examples of analogous electrical and mechanical fractional order system, theorem, Analytical dynamics of fractional type discrete system.

Abstract

This contribution is a new integral and original composition of the previous published or presented author's results.

First, a number of fractional order type, standard light visco-elastic elements are used for coupling between a numbers of mass particles for modeling a fractional order discrete system with finite number of degrees of freedom of motion.

Second, discrete continuum fractional type layers, consisting by standard light visco-elastic fractional type elements, are used for coupling a numbers of deformable bodies (beams, plates or membranes) in hybrid system.

Previous defined two classes of fractional type systems, one discrete and second discrete continuum systems are analyzed and some analogies are identified.

For energy analysis, we start with determination of the generalized functions of fractional order dissipation of mechanical energy for each of the standard light visco-elastic fractional order type elements in both of the systems. The generalized functions of fractional order dissipation of mechanical energy is founded and previously published by author of this paper.

Second, a model of fractional order oscillator with one degree of freedom is presented and corresponding kinetic parameters in free and forced regimes are analyzed as basic models of the independent normal fractional order oscillators and modes of a class of the fractional order type discrete system with finite number of degrees of freedom and

of eigen time functions in one eigen amplitude form of discrete continuum fractional order multi-deformable-body system transversal oscillations.

Third, for a class of the discrete system dynamic with finite number of degrees of freedom, and fractional order dissipation of energy of the system of independent eigen main coordinates, and as well as corresponding independent eigen main modes in free and also in forced oscillatory regimes, are defined.

Fourth, starting from matrix fractional order differential equation of defined class of the system dynamic with finite number of degrees of freedom, and fractional order energy dissipation, relation between total mechanical energy (sum of kinetic and potential energies) and generalized function of fractional order energy dissipation is presented. Also, using formulas of transformation of a system of independent generalized coordinates and eigen main coordinates of considered class of fractional order system dynamics relation between total mechanical energy (sum of kinetic and potential energies) and generalized function of fractional order energy dissipation on one eigen main fractional order mode is presented. On the basis of these relations, two theorems of energy fractional order dissipation of this class of the fractional order system with finite number of degrees of system are defined and proofed.

A number of energy change theorems are defined.

At end, based on the previous listed results, a new branch of analytical mechanics, as an Analytical dynamics of fractional type discrete system is founded.

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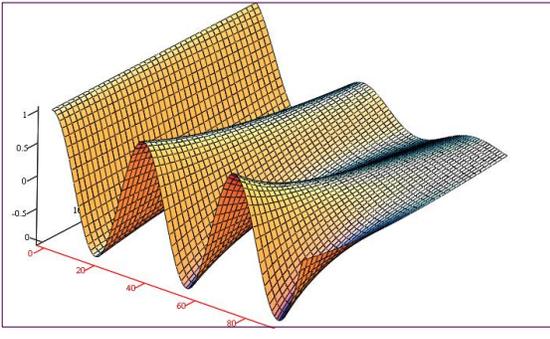


Figure 1.a* Modes of like cosines

$$T_{s,\cos}(t, \alpha) = \sum_{k=0}^{\infty} (-1)^k \omega_{(\alpha)s}^{2k} t^{2k} \sum_{m=0}^k \binom{k}{m} \frac{\omega_{(\alpha)s}^{-2m} t^{-cm}}{\omega_s^{2m} \Gamma(2k+1-cm)}$$

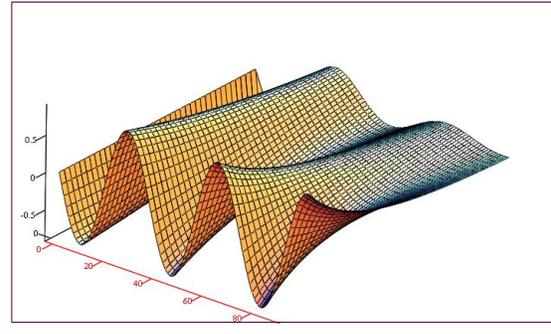


Figure 1.b* Derivative of modes of like cosines

$$\mathcal{R}_{\cos}(t, \alpha) = \sum_{k=1}^{\infty} (-1)^k \omega_{\alpha}^{2k} t^{2k} \sum_{m=0}^k \binom{k}{m} \frac{(2k-cm)\omega_{\alpha}^{-2m} t^{-cm}}{\omega_o^{2m} \Gamma(2k+1-cm)}$$

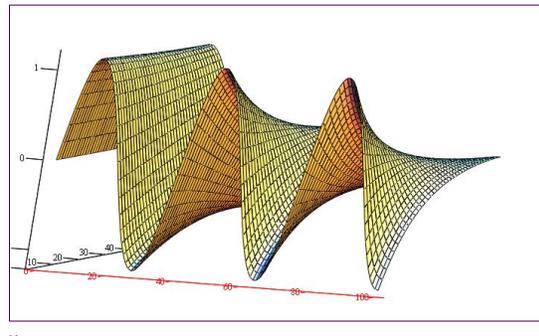


Figure 2.a* Modes of like sines

$$T_{s,\sin}(t, \alpha) = \sum_{k=0}^{\infty} (-1)^k \omega_{(\alpha)s}^{2k} t^{2k+1} \sum_{m=0}^k \binom{k}{m} \frac{\omega_{(\alpha)s}^{-2m} t^{-cm}}{\omega_s^{2m} \Gamma(2k+2-cm)}$$

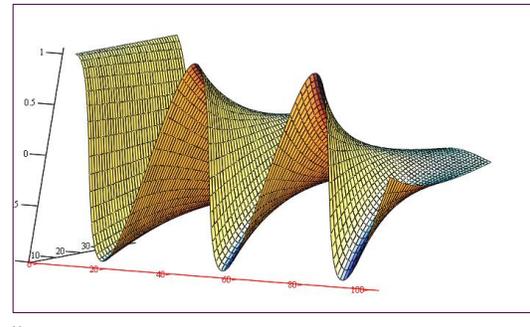
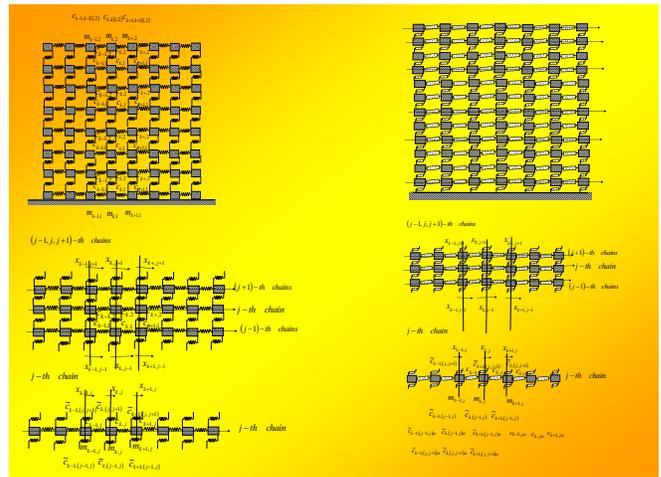
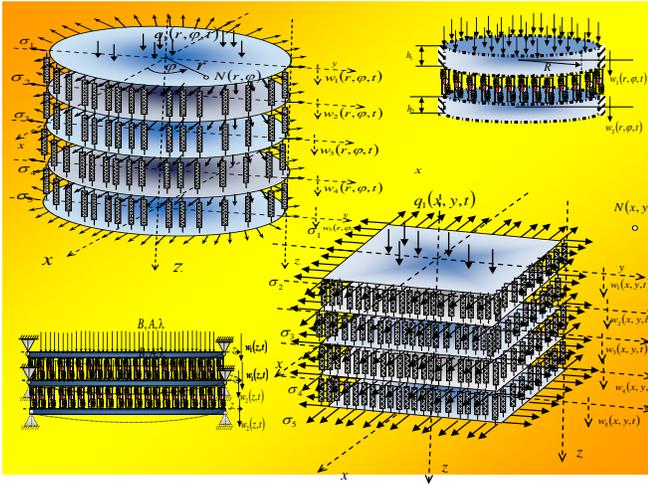


Figure 2.b* Derivative of modes of like sines

$$\mathcal{R}_{\sin}(t, \alpha) = \sum_{k=0}^{\infty} (-1)^k \omega_{\alpha}^{2k} t^{2k} \sum_{m=0}^k \binom{k}{m} \frac{(2k+1-cm)\omega_{\alpha}^{-2m} t^{-cm}}{\omega_o^{2m} \Gamma(2k+2-cm)}$$



Electromechanical analogy between electrical and mechanical fractional order system dynamics with two degrees of freedom (fractional order systems with non-holonomic constraint)

$\dot{x} = \dot{x}_1 + \dot{x}_2$ non-holonomic constraint

$F_{13} = -m_3 \ddot{x}_3 = m_3 (\ddot{x}_1 - \ddot{x}_2)$

Schematic model of mechanisms for realization of non-holonomic constraint

APPLIED MECHANICS REVIEWS

Application of Fractional Calculus for Dynamic Problems of Solid Mechanics: Novel Trends and Recent Results

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The present state-of-the-art article is devoted to the analysis of new trends and recent results carried out during the last 10 years in the field of fractional calculus application to dynamic problems of solid mechanics. The review involves the papers dealing with study of dynamic behavior of linear and nonlinear (DDE) systems, systems with non and nonlocal BVPs, as well as linear and nonlinear systems with an arbitrary number of degrees of freedom, vibrations of rods, beams, plates, shells, membrane combined systems, and multibody systems. Various aspects of the considered field are also presented as well. The results obtained in the field are critically examined in the light of the present state-of-the-art and state-of-the-art in the fractional calculus in engineering problems and practice. This article reviews 172 papers and involves 27 figures. [DOI: 10.1155/4000965]

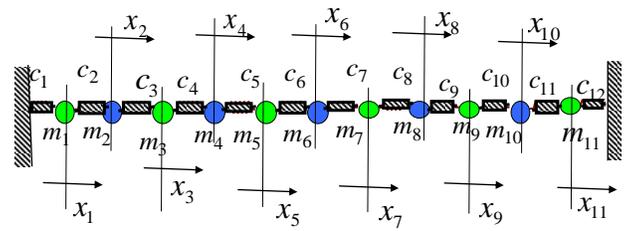
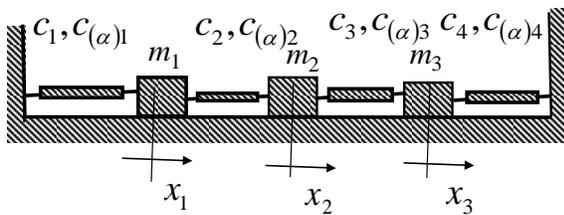
Keywords: fractional integrodifferentiation, free vibrations of viscoelastic systems with finite and infinite number degrees of freedom, impact response

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Towards Energy-Efficient Reliable On-Chip Communications based on LDPC Codes

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Keywords

Fault-tolerance, LDPC codes, iterative decoding, noisy decoders, stochastic resonance, timing-errors.

Summary

Due to huge density integration increase, lower supply voltages, and variations in technological process, complementary metal-oxide-semiconductor (CMOS) and emerging nanoelectronic devices are inherently unreliable. Moreover, the demands for energy efficiency require reduction of energy consumption, which can be done by aggressive supply voltage scaling. The voltage reduction causes data-dependent timing-related errors and it is widely accepted that future generations of circuits and systems must be designed to deal with such errors.

Over the past years, there has been a surge in interest in error control schemes that can ensure fault-tolerance in unreliable hardware. The most popular class of codes resilient to logic gate faults are low-density parity-check (LDPC) codes [1-7]. Their attractiveness lays in the theoretical guarantee that the decoding hardware overhead required to ensure reliable operation grows only linearly with the code length even when logic gates are faulty. Such fault tolerant decoders are based on hard-decision message-passing and bit-flipping decoding algorithms, which unlike more complex algorithms, limit the error propagation in a decoder caused by faulty logic gates.

Our recent work was aimed to answer several fundamental questions as: (i) what effect logic gate failures have on performance of decoders of LDPC codes, (ii) is it possible to guarantee correction of channel errors if unreliable logic gates are used in the decoding process and (iii) is possible to construct fully reliable memory prone to data-dependent failures. The answer to the first question is not trivial in the light of new discoveries [3-4], which reassess the intuitive conclusion that unreliability always leads to performance degradation. We show that data-dependent gate failures can have positive impact on Gallager-B decoding algorithm applied on quasi-cyclic LDPC codes, and reduce the residual error level.

Lack of constructive LDPC code designs, which guarantee correction of a certain number of channel

errors, makes error correction analysis of LDPC codes one of the most significant open problems. Only a few practically significant decoders, made of reliable hardware, can guarantee correction of a fixed fraction of channel errors. We have shown that a subclass of LDPC codes, named expander codes, can correct a fixed fraction of channel errors, even when the decoder is made from unreliable logic gates [6]. In addition, we have shown that the most reliable memories store information in a form of codewords of expander codes [7].

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Decentralized Clustering Algorithm over Compressed Data

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Keywords

Clustering; Decentralized Algorithm; Data Compression

Summary

Networks of sensors are now used in a wide range of applications in medicine, in telecommunications, or in environmental domains [1]. In these applications, a fusion center can collect all the data from all the sensors and perform a given estimation or learning task over the collected data. However, it is not always practical to set up a fusion center, and as an alternative, the sensors may perform the learning task by themselves in a fully decentralized setup. In such a decentralized setup, each sensor must perform the learning with only partial observations of the available compressed measurements.

Here, we assume that the sensors have to perform decentralized clustering on the data measured within the network. The aim of clustering is to divide the data into clusters such that the data inside a cluster are similar with each other and different from the data that belong to other clusters. Clustering is considered in various applications of sensor networks, such as parking map construction [2] or controller placement in telecommunication networks [3]. One of the most popular clustering algorithms is K-means [4], due to its simplicity and its efficiency. However, the major drawback of K-means is that it requires the knowledge of the number of clusters K prior to clustering. When K unknown, it is possible to apply a penalized method that requires applying the K-means algorithm several times with different numbers of cluster [5].

In a network of sensors, it is of high importance to reduce the sensors energy consumption in order to increase the network lifetime. In such context, most of the sensors energy consumption is due to information transmission via the communication system. It is thus crucial to lower the exchange of information between sensors, by relying on data compression and by designing the decentralized learning algorithm so as to minimize the amount of data that has to be exchanged between sensors. In this sense, the decentralized K-means algorithm of [6] must be repeated several times when K is unknown, which dramatically increases the amount of data exchanged within the network.

In this work, we introduce a novel decentralized clustering algorithm that reduces the sensors energy consumption by addressing the above two issues. First, the pro-

posed algorithm, called CENTRE-X, works directly over compressed data, which avoids complex decoding operations prior to clustering. Second, it does not require the prior knowledge of K , which greatly reduces the amount of data that needs to be exchanged between sensors. Unlike K-means, our algorithm models the data collected by the sensors as a centroid corrupted by Gaussian noise, and it assumes that the noise covariance matrix is known. While in K-means, the estimation of K requires data exchange between sensors, the noise covariance matrix can be estimated on the fly by each sensor, via a bunch of parametric, non-parametric, and robust methods, see [7, 8].

In this presentation, we show that the proposed decentralized algorithm greatly reduces the amount of data exchange between sensors, while maintaining a clustering performance equivalent to K-means.

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Rate-adaptive LDPC Code Construction for Free-Viewpoint Television

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Keywords

Free-Viewpoint Television; Slepian Wolf source coding; LDPC codes; Rate-adaptive codes; Protographs

1. Introduction

Free-Viewpoint Television (FTV) is a system for watching videos in which the user can choose its viewpoint freely and change it instantaneously at anytime [1]. For instance, in a football game, a user may decide to follow a player or to focus on the goal. A practical FTV system requires to store all the views of the video on a server which should be able to handle a large number of users.

In this system, each user can request to the server a random subset of the views of the video. In order to reduce the amount of transmitted data from the server to the user, we would like to exploit the fact that the previously requested views are still available when the current view is decoded by the user. This can be represented as a problem of source coding with side information available at the decoder, where the current requested view is the source X and the previous requested view is the side information Y [2], see Figure 1. However, the statistical correlation between the source and the side information varies depending on the previous user request. Therefore, the coding rate must be adapted on the fly depending on the previous request.

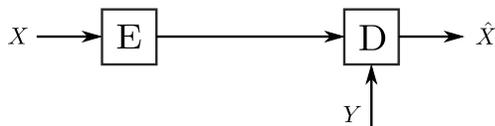


Figure 1: Slepian-Wolf Source Coding

Low-density parity-check (LDPC) codes were first introduced for channel coding [3], but they can also be used for the problem of source coding with side information at the decoder, as proposed in [4]. In this paper, we propose a novel LDPC code construction for the problem of source coding with side information. Our code construction allows the server to adapt the coding rate on the fly, depending on the previous request of the user.

2. Proposed Method

To construct good LDPC codes for source coding, we can combine different methods. A protograph [5] is a small Tanner Graph that represents connections between different types of variable nodes and check nodes. An LDPC code can be generated from a protograph by repeating the protograph structure, and by interleaving the connections between the variable nodes and the check nodes of the corresponding types, see Figure 2 for an example. The performance of an LDPC code highly depends on its underlying protograph, and in the case where no rate-adaptation is needed, several methods were proposed to optimize the code protograph [3].

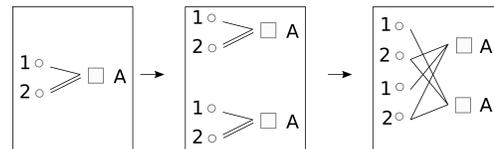


Figure 2: Construction of an LDPC code from a protograph. Left figure is the initial protograph. Middle figure is the protograph duplication. Right figure is the interleaving.

When rate-adaptation is required, several standard rate-adaptive code construction methods can be applied [6, 7]. The most standard one is the Low Density Parity Check Accumulated (LDPCA) construction [7] which permits to obtain low rate codes from an initial high rate code. LDPCA construction combines several lines together in order to construct lower rate codes. However, it does not leave the choice of line combinations (accumulated structure) and bad combinations can generate a lot of short cycles. As short cycles may highly degrade the code performance, we propose a new rate-adaptive construction that limits the number of short cycles.

In this method, we choose line combinations that add the least number of cycles. In this way, we generate a sequence of rate-adaptive codes that perform better than LDPCA. In addition, since the code protograph can help us choose lines combinations that improve the convergence of LDPC decoders, we propose a method that can optimize the protographs for all the considered rates.

Our final rate-adaptive LDPC code construction combines protograph optimization at all the considered rates and great reduction of the number of cycles in the constructed codes. It shows a great performance improvement of up to an order of magnitude compared to LD-PCA.

3. Acknowledgement

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Optimization of ultra-reliable low-latency communication in 5G mobile networks

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Keywords

Reliability; Latency; Communication-information theory

Summary

The use case scenarios for the fifth generation (5G) mobile wireless networks include ultra-reliable low-latency communications (URLLC) with strict requirements, especially in terms of latency and reliability, enhanced mobile broadband (eMBB), addressing human-centric use cases for access to multimedia content, services and data, and massive machine type communications (mMTC) for a very large number of connected devices typically transmitting a relatively low volume of non-delay-sensitive data. Among the three service categories, design of the ultra reliable and low-latency communication (URLLC) service is the most challenging [1]. **Reliability** is defined as success probability of transmitting a predefined number of bytes within a certain delay. **Latency** is defined as the delay a packet (containing a certain number of data bits) experiences from the ingress of a protocol layer at the transmitter to the egress of the same layer at the receiver. After more than five years of research activities in 5G mobile communication system, development and standardization phase has showed that URLLC with strict requirements is accepted scenario. In this work the fundamental tradeoffs among reliability, latency and throughput in 5G networks are outlined. The important results is that there is a need to optimize the transmission of signaling information based on new views on classical communication-information theory model.

A. The tradeoff among reliability and latency

Current cellular systems are very complex with many different elements contributing to the reliability, latency and throughput. It is challenging to fulfill simultaneously stringent requirements. The price to pay for improvement in throughput and reliability is the degradation in terms of latency (Fig.1a). There are three main approaches to address the problem: analytical models, semi-analytical models and simulations. Analytical results provide valuable insight to the tradeoffs among reliability and latency parameters. The **effective bandwidth** function is defined as the minimum service rate necessary to deliver the data by fulfilling certain latency requirements. Analogously, the **effective capacity** is

defined as the maximum constant arrival data rate that a given time-varying service process can support while meeting the delay constraint (Fig.1b). In both cases, a high value of the parameter ν indicates a more severe delay requirement. Therefore, the effective bandwidth curve of a traffic source increases with ν , starting always at the source mean rate and tending towards the peak rate of the source as $\nu \rightarrow \infty$. On the other hand, the effective capacity of the channel starts at Shannon's capacity when $\nu=0$, with no delay constraints imposed, and decreases asymptotically with ν .

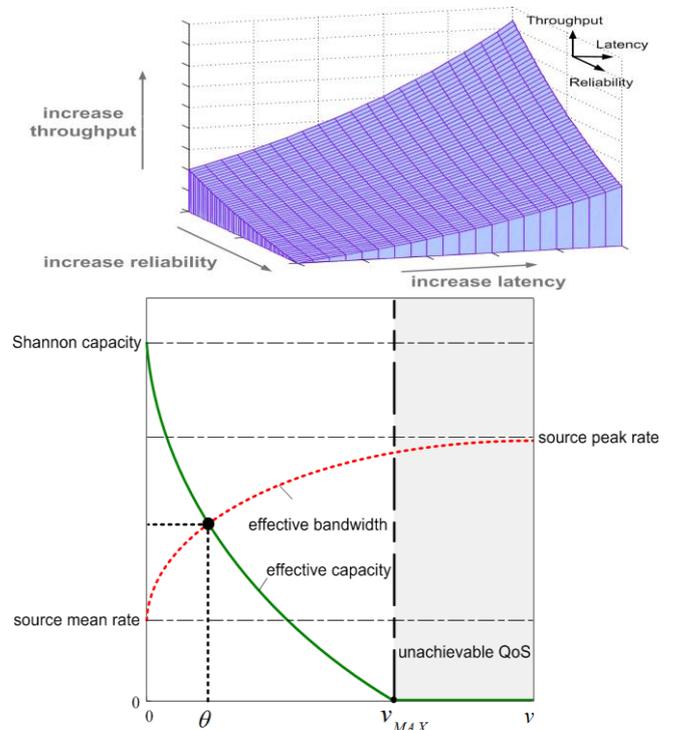


Fig. 1 a) The fundamental tradeoffs among throughput and latency–reliability, b) effective bandwidth and capacity in function of ν (delay requirement) [2].

A **working point** of the system Θ can be defined as the intersection of the two curves. The joint use of the effective bandwidth and capacity functions in a semi-analytical framework enable fundamental investigation of the tradeoffs among reliability and latency. Statistically reliable simulation results of the 5G system are partially used as an input to the analytical expressions [2]. The model is capable of estimating the QoS metrics in different scenarios. The following candidate techniques are identified for 5G

system improvements: the increased diversity, advanced interference management, multi-cell baseband pooling, and shorter sub-frame duration for a reduced transmission delay.

B. New communication-theoretic principles

Models of current cellular systems are very complex which leads to mathematically intractable problems. The alternative is to enable assumptions and limiting the scope of the results. However, traditional assumptions in *Shannon's* information and communication theory require a new view on latency constraints, auxiliary procedures and new stochastic model of very rare events.

The simple *Shannon's* communication model captures the essential stochastic nature of a communication system [3]. The key information-theoretic result is that, given sufficiently long time and sufficiently many communication channel uses, one can obtain almost a deterministic, error-free data transmission whose rate is dictated by the channel capacity. The model is challenged in URLLC in three aspects [4, 5].

1. The number of available channel uses is limited due to the latency constraints. High reliability implies low P_e probability of packet drop/error (with infinite latency). However, the opposite is not necessarily true, as in URLLC it is needed to achieve low probability in a time duration limited by the deadline (Fig. 2).

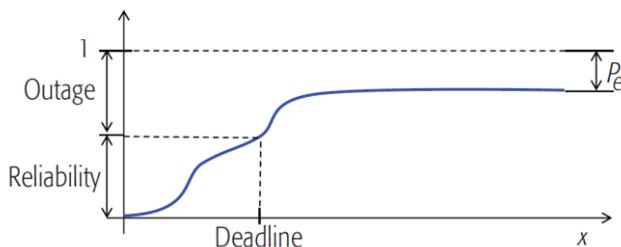


Fig. 2 Reliability as the probability that the latency does not exceed a pre-described deadline.

The used number of available channel is proportional to the product of the time duration and the bandwidth of the transmitted signal. Hence, by increasing the bandwidth, we obtain two advantages: more available channel uses and more frequency diversity. Increasing bandwidth enables decrease the channel use time duration, or to keep the duration fixed and to increase the number of channel uses in frequency.

2. The URLLC communication model transmits the actual data as well as exchange metadata in auxiliary procedures, including channel estimation, packet detection, and additional protocol exchanges. The capacity results of *Shannon's* information theory implicitly assume that when data are

transmitted to receiver, both sides know that the transmission is taking place as well as when it starts and ends. In practice, this information needs to be conveyed through transmission of metadata (control information) which size is much smaller than the data, as well as the amount of resources (channel used) spent on sending metadata is negligible. However, this does not hold in URLLC, since the data size is often small and comparable to the metadata size, and one explicitly needs to optimize the coding/transmission of metadata. Further, considering the high reliability levels in URLLC one can no longer assume that the metadata transmission, as well as all auxiliary procedures, are perfectly reliable.

3. URLLC requires that the model is considered in regimes of very rare events in optimization wireless design and performance evaluation. Classical *Shannon's* stochastic models accurately captures the statistics of all relevant stochastic factors. However, the challenge of URLLC is that they require modeling of factors occurring very rarely (e.g., with probability of 10^{-6}) within the packet duration, if the target reliability is higher (e.g., outage probability in the same period $<10^{-7}$). Therefore, proper stochastic models of the wireless environment are crucial in design of packets and access protocols for diversity techniques making URLLC affordable.

The future research directions has to be build on detailing the optimal design of the building blocks [6, 7] and combine them toward a complete URLLC solution that corresponds to a use case (VR, automotive, Industry 4.0, Smart Cities).

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Ant Colony Optimization Applied to the Blocks Relocation Problem

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Keywords

maritime shipping; blocks relocation problem; heuristics; Ant Colony Optimization

Summary

In the recent decades an unprecedented growth of international trade has occurred. The vast majority of it is carried out by the international shipping industry through container terminals. These terminals represent gigantic logistic centers which serve as transshipment points. Their main function is to serve as temporary storage points, which are used for unloading containers from very large transport vessels and transferring them to smaller vessels, vehicles or trains for further distribution, and in the opposite direction. One of the main issues at container terminals is the limited space used for storage. As a consequence, containers are piled up at the container yard in such a manner to increase the space utilization, more precisely by using block stacking [14]. The loading and unloading operations are directly related to the stacking (decreases the number of moves of containers) and indirectly to the horizontal container transport. The speed of these operations is also essential for the length of time that a vessel needs to spend docked, and is a standard measure of efficiency of a port. Because of this a significant amount of research has been dedicated to the problem of minimizing the number of relocations of containers inside the port. For this practical problem several different models have been developed like the Blocks Relocation Problem (BRP), the Re-Marshalling Problem (RMP), i.e. intrablock marshalling and the Pre-Marshalling Problem (PMP) [2, 19].

In the recent years a noteworthy research effort has been dedicated to the BRP due to the fact that it effectively model practical problems related to the loading operations at container terminals. The basic model has been extended in several ways to better the modeled real-world problem with the used of different objective functions [15, 28, 12], sets of constraints [3, 8, 28], three dimensional yards [15, 9], an on-line version [24, 21, 1],

existence of uncertainties [1, 26], etc. Although most of these models increase the complexity of the BRP, the basic concept for solving them are taken from methods developed for the original BRP. Several methods have been developed for solving the BRP to optimality (integer programming [24], branch-and-bound [20] and A*[29, 8] algorithms). Due to the NP-hardness of the problem [3] a wide range of methods have been developed for finding near optimal solutions for the BRP (greedy algorithms and variations [27, 16, 25, 23, 13], a tree search [10], a domain-specific knowledge-based algorithm [8], [25, 17], the corridor method [4], genetic algorithms [11], etc.)

In this talk, the focus is on implementing the ant colony optimization (ACO) [7] to the BRP. In case of problems related to loading/unloading operations of containers, ACO has only been applied to the PMP [22]. One of the main reasons for the lack of research in this direction is the complexity of defining the pheromone matrix since the commonly used heuristic functions are related to different states of the bay whose number is enormous. In the work of Tus et al. [22], in case of PMP, this problem has been resolved by the dynamic allocation of the pheromone matrix. One drawback of this approach is the loss of simplicity of implementation of the ACO method, which is one of its main advantages. To avoid this issue, we have used an alternative formulation of the list of candidates that is used in the related greedy algorithms. Further, a new direct heuristic for the BRP is defined which is suitable for use in the ACO transition rule. Finally, a novel approach to defining the pheromone matrix is presented. The main idea is to have the matrix in form of a multidimensional array, having a small number of dimensions, that only stores a small but important amount of information about the bay state.

The method has been applied for both the restricted (rBRP) and unrestricted version (uBRP) of the BRP. In developing the ACO algorithm, a new, simple to implement, heuristic function has been proposed for solving the uBRP that manages to outperform similar existing approaches. Further, a new formulation of the

standard greedy algorithm for the rBRP is introduced, which has the advantage that it can be directly applied to the uBRP. The proposed greedy algorithm has been extended to the ACO metaheuristic. In developing this approach a novel concept has been introduced for defining the pheromone matrix that only contains partial information about the problem. Our computational results show that the proposed ACO algorithm manages to outperform the current state-of-the-art (tree search [10] and domain-specific knowledge-based algorithm [8]) in both computational cost and quality of found solutions. Further, we have shown that the proposed ACO algorithm is highly robust in the sense that it can be extended to alternative objective functions for the BRP.

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An Application of Statistical Depth in Clustering of High-dimensional Acoustic Data

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Keywords

Tukey depth; Tukey median; big data; data science; un-supervised learning

Summary

In order to cluster a large set of data, statistical procedures use comparison of the data by the size of one or more characteristic parameters. The classical mean/variance paradigm fits nicely into a setup with Gaussian distributions, but the results can be sensitive on small deviations from the assumed model, i.e. mean-based algorithm such as k-means lack robustness. To improve robustness, statisticians use alternative measures of centrality, like medians in scalar data. In a multidimensional setup we can start with so called statistical depth functions and define a median set as a set of deepest points.

In this paper, we introduce a new approach for clustering large set of high-dimensional data, called ABClustering, based on ABCDepth algorithm for finding multivariate median [1]. ABClustering takes three steps and each step will be described shortly below.

0.1. Step one - Analyze the data

The initial and the most important task in every data science work is to understand the data and to understand the problem that should be solved. Knowing that, given data set can be modified in the most appropriate way. We analyze acoustic data set that contains probability density functions of frequency dependent angular distributions for external noise incident energies.

Noise incident energies are taken from $l = 12$ locations, L_i , $i = 1 \dots l$ and each location is described with $n = 10$ continuous functions at a certain frequency band. Furthermore, each of those continuous functions are discretized into $m = 91$ noise incidence angles, so each L_i can be represented as a matrix $m \times n$:

$$L_i = \begin{pmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,m} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,m} \end{pmatrix}$$

i.e.

$$L_i = \{\mathbf{X}_j\}, j = 1, \dots, m, \mathbf{X}_j = \{x_{k,j}\}, k = 1, \dots, n.$$

Each location represents a different type of a street - some streets are wide, some of them are narrow, streets are bordered with high-rise or low-rise buildings, parking lots or trains can be close to the streets, streets are more or less busy etc. Every of those parameters has an influence on noise incidence energies. The aim is to cluster those locations, i.e. to find the way they make clusters that relies on locations' similarities. Based on locations similarities, a proper facade noise isolation can be found for each location type (cluster).

The input of ABClustering algorithm is L_i matrix, i.e. a set of L_i matrices.

0.2. Step two - Dimensionality reduction and median calculation

Clustering high-dimensional data is a very demanding task. Some approaches for clustering high-dimensional data and its summaries can be found in [13], [14].

Distances based clustering is a well known approach and for the most algorithms, the distances are calculated between data points from the given data set or between data point from the data set and the data set mean value [15]. Instead, ABClustering algorithm uses median value to get the distances between data points and median point. That way, each multidimensional point is represented by its distance from median value and the multidimensional data set is reduced to one dimension.

The approach of dimensionality reduction is based on ABCDepth algorithm [1] for approximate calculation of multivariate median. ABCDepth algorithm relies on lo-

cation depth or Tukey depth ([12], [10], [11]). According to this depth function, a depth of a point is defined as the minimum number of sample points on one side of a hyperplane through the point. In [1] this definition is extended to:

Definition 0.1 (section). *Let \mathcal{V} be a family of convex sets in \mathbb{R}^d , $d \geq 1$, such that: (i) \mathcal{V} is closed under translations and (ii) for every ball $B \in \mathbb{R}^d$ there exists a set $V \in \mathcal{V}$ such that $B \in \mathcal{V}$. Let \mathcal{U} be the collection of complements of sets in \mathcal{V} . For a given probability measure μ on \mathbb{R}^d , let us define*

$$D_{\mathcal{V}}(x; \mu) = \inf\{\mu(U) \mid x \in U \in \mathcal{U}\} = 1 - \sup\{\mu(V) \mid V \in \mathcal{V}, x \in V'\}.$$

The function $x \mapsto D(x; \mu, \mathcal{V})$ will be called a depth function based on the family \mathcal{V} .

This theorem is derived from [4].

Unlike all other approximate algorithms (see [5], [6], [7], [8], [9]), ABCDepth does not need to calculate projections of sample points to directions. Instead, it calculates multivariate median based on convex sets intersection in \mathbb{R}^d . In ABCDepth, convex sets are balls since they are easy for implementation. Level sets (or depth regions or depth-trimmed regions) can be defined as:

$$S_{\alpha}(\mu, \mathcal{V}) = \bigcap_{B \in \mathcal{V}, \mu(B) > 1-\alpha} B.$$

Tukey median is a point or a set of a points maximizing Tukey depth, i.e. S_{α} with the highest alpha.

Described approach provides linear complexity in d , so overall ABCDepth complexity is :

$$O((d+k)n^2 + n^2 \log n),$$

where k is a number of iteration and its asymptotical upper bound is $\frac{n}{2}$ (see Remark 3.2. in [1]).

This complexity provides a great advantage over all other approximate algorithms.

Using ABCDepth, we calculate median for \mathbf{X}_j vectors from each L_i matrix:

$$\text{med}_j(\{L_1(X_j), L_2(X_j), \dots, L_i(X_j)\}), \quad i = 1, \dots, 12 \text{ and } j = 1, \dots, m.$$

After that, for each \mathbf{X}_j from L_i matrix, the algorithm calculates the distance between X_j and median med_j :

$$\text{dist}(L_i(X_j), \text{med}_j) = d_{ij}.$$

That way, we have m one-dimensional data points, instead of m points of dimension n , and each X_j is represented with its distance:

$$L_i = \{d_{i1}, d_{i2}, \dots, d_{ij}\}, \text{ where } j = 1, \dots, m.$$

0.3. Step three - Clustering of distances

ABClustering algorithm uses [2] function to cluster distances calculated in the previous step. Iteratively, algorithm groups distances d_{ij} from each matrix L_i . The number of iterations is equal to m (to the number of angles of noise incidence). At the end of each iteration, there are maximal l (number of locations) clusters, in the case if each location belongs to the different cluster. In other words, algorithm groups $\{d_{11}, d_{21}, \dots, d_{121}\}$ distances in the first iteration, $\{d_{12}, d_{22}, \dots, d_{122}\}$ distances in the second iteration, etc. In each iteration, one location belongs to exactly one cluster, but it does not mean that the same location can not belong to same other cluster in different iteration. In ABClustering algorithms, clusters obtained from described process are called: clusters of type C .

After m iterations, the algorithm counts how many times location L_i appeared in the same cluster of type C with some other location. That way, ABClustering does re-clustering of locations (those new clusters are called clusters of type C') in the following way: location, L_x , $x \in \{1\dots 12\}$ is placed in a new cluster of type C' with some other location L_y , $y \in \{1\dots 12\} \setminus x$ iff L_x appeared most times with location L_y in clusters of type C . In that case, we say that L_x and L_y are in relation: $L_x \rho L_y$.

Clusters of type C' are in a form of connected components of a weighted graph whose reachability is an equivalence relation. According to its transitive property:

$$(L_x \rho L_y) \wedge (L_y \rho L_z) \implies L_x \rho L_z.$$

Each connected component represent one cluster of type C' . Nodes represent locations. Extraverted edges shows how many times locations L_x and L_y appeared with each other in clusters of type C . For example, in Figure 1, locations M and MV appeared the most (28) times with each other in clusters of type C . Directed edges from location L_x to location L_y show how many times location L_x appeared in the same cluster of type C with location L_y . For example, location E appeared the most (23) times with location M. Due to the transitive property explained above, locations M, MV and E make one cluster of type C' .

An application of ABClustering algorithm with detailed data set description can be found in Miloš Bjelić PhD thesis [3], section 5.4.

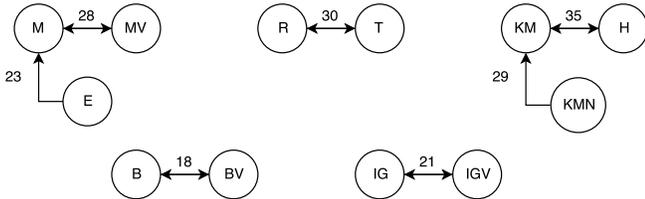


Figure 1: Clusters view in a form of connected components of a graph

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Computing the inverse and pseudoinverse of time-varying matrices by the discretization of continuous-time ZNN models

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Keywords

Zeroing neural network; Inverse matrix; Moore-Penrose inverse; Multi-step methods

Summary

We consider discretizations of continuous-time Zhang Neural Network (ZNN) for computing time-varying matrix inverse and/or pseudoinverse. These discretizations incorporate scaled Hyperpower methods as well as the Newton method. We apply the most general linear multi-step method based scheme, including all known discretization schemes. Particularly, 4th order Adams-Bashforth method based scheme is proposed and numerically compared with known iterative schemes. In addition, the ZNN model for matrix inversion is extended to the pseudoinverse computation. Convergence properties of these extensions are also investigated.

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Self-organized criticality in social dynamics of knowledge creation

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Summary

Methods of Statistical Physics for studying complex systems in the physics laboratory have been widely used for the interpretation of the vast amount of social dynamics data collected on various Websites, which has provided a unique opportunity for the quantitative study of social phenomena in analogy with natural sciences. In particular, the physics theory of interacting nonlinear systems which are driven far from equilibrium play a crucial role in understanding the mechanisms that enable the emergence of collective social phenomena on the Web. In this lecture, we discuss some challenges of physics approaches to social dynamics and illustrate them by considering a specific type of data from social processes of knowledge creation via Q&A site Mathematics and IR Chats Ubuntu as well as the appropriate agent-based modeling. We show the types of networks that emerge in these processes and underly the dynamics while keeping the emphasis on the features of self-organized criticality of these stochastic processes.

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How random are complex networks?

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Keywords

Complex networks, random graphs, topology

Summary

Every complex system can be represented as a complex network, where constituent units are represented with nodes and interactions between them are expressed by network links [1]. These networks are neither of regular or random structure, but rather an intricate combination of order and disorder. Scientists have developed large set of different topological measures for characterization and description of different structural properties of real networks [2]. It turns out that these statistical measures are not independent, i.e., many properties appear as a statistical consequence of relatively small number of fixed topological properties in real network. Here we explore this dependence by applying the method of dk-series to six real networks - the Internet, US airport network, human protein interactions, technosocial web of trust, English word network, and an fMRI map of the human brain - representing different complex systems [3]. We find that many important local and global topological properties of

networks are closely reproduced by dk-random graphs with the same degree distribution, degree correlations, and clustering as in the studied real network. We discuss important conceptual, methodological, and practical implications of this evaluation of network randomness.

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Some fuzzy set (in)equations in case of a complete codomain lattice

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Keywords

fuzzy set inequations; fuzzy set equations; complete lattice; monotonous operator; transitive closure.

Summary

Some fuzzy set equations and inequations that are often considered are those in which we have some kind of composition of an unknown fuzzy set with the given fuzzy relation on one side of the (in)equation and the same fuzzy set on the other side of the (in)equation. The solution of such equations are called eigen fuzzy sets (see e.g. [5]). Definitions of the composition of a fuzzy set with a fuzzy relation vary depending on the class of the codomain lattice of the considered set of fuzzy sets. We shall consider the case of the complete codomain lattices, considered also in previous researches ([4], [3]). This is quite general case, knowing that there are many more special cases, such as residuated lattices (see e.g. [2]), or even $[0, 1]$ -interval.

Some fuzzy set equations and inequations, as well as their systems, are of the form $\Phi(u) = u$, $\Phi(u) \leq u$ and $\Phi(u) \geq u$, where Φ is a monotonous operator. Therefore, we may prove and apply some properties of monotonous operators on the complete lattices. Among the inequations of the form $\Phi(u) \leq u$ are the one having transitive relations for its solutions, while among the equations of the form $\Phi(u) = u$ are those having eigen fuzzy sets for its solutions.

Using properties of a monotonous operator on the complete lattices, we shall be able to generalize a result by Jimenez et al. from 2011 regarding the existence of the solution of the equation $\mu \circ R = \mu$ in a set defined recursively (Theorem 7 in [3]). We shall likewise prove the

existence of the greatest "exactly transitive" relation contained in a given transitive relation, and the existence of solutions to some systems of fuzzy equations.

We prove that a version of a general algorithm for computing the transitive closure (see [1]) proven to work in a special case of meet-continuous codomain lattices [6] does not work here, in general. However, a sufficient condition under which we may construct solutions of the fuzzy inequations of the form $\Phi(u) \leq u$, where Φ is a monotonous operator, is given here. This gives some of the existing results in case of the meet-continuous codomain lattices (see [6]) as special cases.

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